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High temperature superconductivity—current status, our theoretical and experimental work

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Abstract : The current status of the theoretical efforts towards understanding the microscopic mechanism behind the phenomenon of high-temperature superconductivity is reviewed. Our Theoretical mechanism based on charge fluctuations is described and workon the magnetic property in the insulating phase as well as in the doped phase are discussed. Some experimental work involving positron annihilation technique is highlighted.

Keywords : High temperature superconductors, charge fluctuations, magnetic property in insulating and doped phase, positron annihilation.

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1. Introduction

The recent discovery of superconductivity at high temperature especially at liquid Nitrogen temperature and above (Bednorz and Müller 1986, Chu *et al* 1987) in the layered copper oxide perovskites has stimulated a lot of theoretical efforts along various different directions to search for the microscopic mechanism responsible for this novel phenomenon. The theoretical approaches can broadly be divided in two main catagories :—(i) those describing the normal phase as a Fermi-liquid and viewing the superconducting transition as a pairing of two Fermionic quasiparticles like in the BCS-approach and (ii) those considering the normal phase as a non-Fermi liquid and describing the superconducting transition as a charged-Boson condensation.

The models under the pairing approach with normal metallic phase as a Fermi-liquid can again be subdivided on the basis of the nature of interaction causing the pairing. The pairing interaction can be charge fluctuation induced or spin fluctuation induced. The former includes (i) phonon, (ii) charge-transfer (CT) exciton and (iii) plasmon mediated attractive interaction. The later includes (i) antiparamagnon mediated, (ii) antiferromagnetic indirect-exchange induced and (iii) spin bag mediated attraction. The charge-fluctuation based models

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have all predicted the pairing in the conventional *s*-wave channel. The pairing has been predicted also in *p* and *d*-wave channels besides *s*-wave by the spin-fluctuation based models.

The models predicting charged-Boson condensation starting from a non-Fermi liquid have so far been primarily of two types viz. resonating valence bond (RVB) and bipolaronic. The RVB theory (Anderson 1987a, 1987b) proposes a spin-charge decoupling of the single particle excitations in the normal phase which is the doped phase of an antiferromagnetic Mott insulator. This 'RVB metal' has spin-less charge e^+ boson called 'holon' and spin $\frac{1}{2}$ chargeless fermion called 'spinon' as the quasi-particles. The superconducting transition corresponds to a condensation of holons. The original bipolaronic model (Chakraverty *et al* 1987) assumes the normal phase to be the doped state of a Peierls insulator with a strong electron-phonon interaction. This leads to the formation of bipolaron which is a bound pair of the usual quasi-particles polarons in the normal phase. These bipolarons are bosonic in nature, which can undergo condensation leading to superconductivity. This model has been modified later and also new bosonic quasiparticles called bisqueeps have been introduced, which presumably have higher condensation temperature.

There has been a very recent suggestion by Varma and coworkers (Varma *et al* 1989) that the normal phase of these oxide superconductors is a 'marginal Fermi-liquid' which is almost a Fermi-liquid with certain peculiar properties. In this model, the normal phase has no spin-charge decoupling for the single-particle excitations ; however a certain number of bound pairs of these charged fermions are also present besides these lone fermions. Superconductivity arises presumably due to the pairing of these fermions in *s*-wave channel, brought about by the attractive interaction mediated by the CT-exciton. Somewhat similar idea had been proposed by Kulik (1988) sometime earlier.

Let us discuss each of these models in greater details and highlight their significance with regard to various special experimental features observed in the normal as well as in the superconducting phase of these new superconductors.

2. Earlier theories of superconductivity

2.1. *BCS-like theories :*

(i) *Phonon-mediated attraction :*

The theories involving conventional mechanism of phonon exchange have been proposed by Kamimura (1987), Weber (1987), Mattheiss (1987) and others to explain the transition temperature ($T_c \sim 40$K) in the compounds like $La_{2-x}(Sr, Ba)_xCuO_4$. The attempt by Weber (1987) to obtain $T_c \sim 90$K for $YBa_2Cu_3O_7$ using Eliashberg's formalism corresponding to strong electron-phonon coupling however, did not succeed suggesting probably a more important role played by some other mechanism. The observed isotope effect in the La-compounds and almost vanishing isotope effect in Y-compounds (Schrieffer *et al* 1988a) seem to be consistent with this, although truly speaking the value of isotope exponent as well as T_c in the case of very strong electron-phonon coupling in the presence of a large Coulomb correlation is still not known correctly. Calculations of Bulaevskii (Bulaevskii *et al* 1988a, 1988b) using Eliashberg function extracted from tunnelling data explicitly shows the very important role of an electronic charge excitation of energy ~ 100 meV in the 1-2-3 systems. Similar calculations by Marsiglio and Carbotte (Carbotte 1990) also support this. The phonons involved in these proposed models are mostly of Jahn-Teller type, breathing mode type and symmetry breaking type.

(ii) CT-Exciton mediated attraction :

Varma and coworkers (Varma and Schmitt-Rink 1987) have pointed out the importance of Coulomb repulsion V between the electrons (or holes) on near-neighbour Cu and O sites besides the usual very large Mott-Hubbard on-site repulsions U_d and U_p on the Cu and O-sites respectively. On the basis of an earlier calculation of Toyazawa and coworkers (Toyazawa *et al* 1966a, 1966b) they have argued that this V leads to a charge-transfer resonance or CT-Exciton type of particle hole excitation which corresponds to a bonding band to anti-bonding band exciton. In the case of copper-oxide superconductors this exciton produces essentially a localised charge fluctuation $Cu^{+2}O^{-2} \rightarrow Cu^{+1}O^{-1}$, which is almost unscreened. It has a much higher characteristic energy (~ 0.5 eV) than the usual Debye energy (~ 0.03 eV) of the phonons and is thus capable of producing much higher T_c (~ 100K) provided electron-exciton attractive coupling is not very small. Varma and coworkers have suggested this exciton-mediated attraction as the major mechanism for high-temperature superconductivity in these copper-oxide superconductors. Experimentally in infra-red absorption spectrum a peak has been observed near 0.4-0.5 eV in polycrystals but it evolves into an almost constant background in single crystals for the La and Y-compounds (Varma *et al* 1989, Schrieffer *et al* 1988a).

Numerical calculations on finite lattices (Hirsch *et al* 1988, Stephan *et al* 1988, Alascio *et al* 1988) indicate hole binding for 2-band model containing V ; however whether it leads to pairing or clustering, is not clear.

A slightly different model invoking CT-Exciton mediated attraction has been proposed by Tesanovic and coworkers (Tesanovic *et al* 1988). The exciton in their theory can correspond to both intra-band and inter-hand charge fluctuations and in particular contains non-bonding-band to anti-bonding band particle hole excitation besides the earlier mentioned bonding-band to anti-bonding-band excitation. Their model, however assumes U_d and U_p .o be small for carrying out a week coupling analysis. They also examine the possibility of both intra-layer and inter-layer pairing and claim to explain the entire phase-diagram seen experimentally for these systems, including the magnetic and charge-disproportionation regime (observed at large doping concentration or oxygen concentration) besides the occurrence of high-T_c in the superconducting regime.

(iii) Plasmon-mediated attraction

Ruvalds (1987) and Kresin (1987) have suggested the possible important role of electronic plasmons with characteristic energy of the order of 1 eV in producing high-T_c in these systems. They argue that because of low carrier density ($\sim 10^{21}$ cm^{-3}) and presumably also because of quasi-two dimensionality of carrier motion the plasmon modes are well defined even for large wave vectors and can mediate an attractive interaction for wave vectors $q \sim K_F$, unlike the 3-dimensional plasmons which undergo a strong Landau-damping for wave vectors of the order of K_F.

The role of ion-plasma branches in producing pairing attraction leading to high T_c has been analysed by Mahanty and Das (1988).

(iv) Spin-fluctuation mediated attraction :

The observed antiferromagnetism in the insulating compounds La_2CuO_4 (Shirane *et al* 1987) and $YBa_2Cu_3O_6$ (Tranquada *et al* 1988) and the proximity of the superconducting phase to the antiferromagnetic phase has motivated many workers to put forward models invoking magnetic pairing mechanism for superconductivity. Neutron and Raman scattering studies (Birgeneau *et al* 1988, Lyons *et al* 1988) show that appreciable *2d* anti-ferromagnetic short-range spin correlations persist even in the superconducting phase with spin-spin correlation length ξ falling from 200Å above T_N in the undoped material to ~ 10-20Å in the superconducting phase. The models with magnetic mechanism can be divided in two broad groups viz. $U \rangle W$ (localised limit) and $U \langle W$ (itinerant limit) where U is the on-site Mott-Hubbard repulsion and W is the band-width in an effective one-band model relevant to the undoped insulating material. The models proposed by Emery (1987), Tosatti and coworkers (Su *et al* 1987), Doniach and coworkers (Doniach *et al* 1988), Rice and coworkers (Gros *et al* 1987) and Aharony and coworkers (Aharony *et al* 1988) all belong to the localised case whereas the model suggested by Schrieffer (Schrieffer *et al* 1988a, 1988b) belongs to the itinerant case.

Emery considers a 2-band model with Cu and O sites occurring explicitly, almost identical to that of Varma *et al* (1989) and shows that the doped holes (assumed to be situated predominantly on O sites) with kinetic energy of the order of t^2 couples to the spin fluctuations occurring on Cu-sites with a coupling strength also of the order of t^2, where t is the hopping amplitude from Cu-site to the nearest O-site and vice-versa. This leads to an effective attraction between the holes and can produce a BCS-gap of s-wave symmetry and also a high value of T_c. A very similar idea was suggested by Aharony and coworkers (Aharony *et al* 1988) which predict an effective attraction between the holes on O-sites mediated by the spin-fluctuation of the frustrated spins on Cu-sites in the doped phase, producing superconductivity of d-wave symmetry with high magnitude of T_c. A somewhat similar mechanism was proposed by Su *et al* (1987) within a 1-band model, which leads to a p-wave pairing. Variational calculation by Rice and others favours d-wave pairing mediated by magnetic interaction. Doniach (1988) suggests a 2-band model of the kind used for heavy-fermion systems and show-that the spin-fluctuation of the localised Cu-holes existing in a quantum spin-liquid state in the doped phase, leads to a *d*-wave pairing of the injected itinerant holes in the wider O-band. They also attempt to explain the experimentally observed anomalous transport property in the normal phase by the idea of superconducting fluctuations arising from the very small coherence length observed experimentally in the systems.

Schrieffer *et al* (1988a, 1988b) proposed a model assuming the magnetic ordering in the insulating phase to be of a commensurate spin-density-wave (SDW) type occurring due to the nesting property of the Fermi surface in a copper-oxide layer, which leads to a gap (Δ_{SDW}) in the single particle electronic excitation spectra. As holes are introduced by doping, the Fermi-surface loses the nesting property locally and the amplitude of $_{SDW}$ is also reduced locally in the vicinity of the holes. This can cause a pair of holes to be bound inside a spin bag with a reduced amplitude of both SDW and Δ_{SDW} within the bag. This attractive potential leads to a *s*-wave pairing and probably also a large value of T_c.

2.2 *Non-BCS-like theories :*

(i) *Resonating valence bond model :*

The original RVB model proposed by Anderson (Anderson 1987a, 1987b ; Anderson *et al* 1987) ascribed the occurrence of superconductivity in these oxides to be due to the mobility of preexisting singlet pairs which are frozen in the Mott insulating state but become mobile in the doped phase due to creation of vacancies. These pairs in real space which are very similar to the Cooper pairs, form a resonating valence bond structure, believed to be a very good approximation to the actual ground state configuration of any frustrated quantum Heisenberg antiferromagnet with spin $\frac{1}{2}$ in 1 and 2 dimensions (Fazekas and Anderson 1974, Majumdar and Ghosh 1969, Anderson 1973). The undoped La_2CuO_4 and $YBa_2Cu_3O_6$ believed to be Mott insulators were thought to possess a strong antiferromagnetic correlations in a 2D Cu-O plane due to an exchange coupling $J \sim \frac{t^2}{U}$ without any true long-range order seen in a Neel state. This RVB state was conjectured to have a gapless excitation corresponding to a pseudo-Fermi surface of a spin $\frac{1}{2}$ particle which could be a spinon or a meron. The pseudo-Fermi surface was thought to be remaining intact even after doping and was supposed to be behind the mysterious large linear specific heat γT observed experimentally in the superconducting phase. This original RVB model was very similar to a pairing theory ; however, even in the doped phase the hopping of the vacancies has to obey the constraint arising from the strong Mott-Hubbard correlation, which was not taken into account properly. To bring out this non-Fermi liquid nature of the doped phase, Zou and Anderson (1988) used the slave Boson theory which also attempts to show the spin-charge decoupling of the quasi-particles in the normal phase. Making use of the idea of splitting of a hole in a conducting 2D layer into holon and spinon, holon-spinon mutual scattering and inter-layer hopping of a hole, this theory could explain a number of features seen in the experimental data (Anderson 1987a, 1987b) such as linear electrical resistivity aT in the basal plane and resistivity $\frac{c}{T}$ along the c-axis, both in the normal phase, as well as linear specific heat γT in the superconducting phase. The experimentally observed flux quantization in units of $\frac{hc}{2e}$, however, seems to be incompatible with the idea of holon condensation leading to superconductivity. Moreover, the observation of Fermi-edge in recent photo-emission experiment (List *et al* 1988, Katayama-Yoshida *et al* 1988) also points towards a Fermi-liquid nature of the doped phase. This idea of holons and spinons has been further extended by Laughlin (Laughlin 1988) and Witten and coworkers (Chen *et al* 1989) to suggest 'anyon superconductivity' arising from the pairing of charged topological spin vortices. It may be mentionworthy that the proponents of RVB theory now consider holons to have a fermionic character in order to explain the observed Fermi-edge (Baskaran 1990).

(ii) Bi-polaronic model.

The extremely small coherence length ($\xi \sim 10\text{-}20$ Å) observed experimentally (Cava *et al* 1987) in these oxide superconductors and indication of a large electron-phonon coupling ($\lambda \sim 2$) in these systems from theoretical calculations (Weber 1987) motivated Chakraverty and coworkers (Chakraverty *et al* 1987) to propose Bose condensation of certain real-space pairs called squeezed bipolarons or bisqueeps as the mechanism for high temperature superconductivity. Several structural studies (Capponi *et al* 1987) in these systems show that orthorhombic distortion in the basal plane an essential condition for the onset of superconductivity. Spectroscopic studies (Bianconi *et al* 1988) performed earlier had exhibited a strong signal of O and an almost complete absence of Cu^{+3} signal, in the doped phase. Chakraverty and coworkers argue that these experimental observations imply the existence of Bosonic excitations called peroxitons (O^{-}–Cu^{+}–O^{-}) which are strong covalently bonded O-O pairs, stabilized through coupling with structural distortion. These stabilized peroxitons are the bisqueeps which can have low effective mass (high mobility) and hence large Bose-condensation temperature. Kulik in his model (Kulik 1988) has also proposed the existence of similar bosonic objects to explain the anomalous properties in the normal phase.

2.3. Superconductivity by pairing from a marginal Fermi-liquid :

Very recently Varma and coworkers (Varma *et al* 1989) from a purely phenomenological approach have suggested a form of the dynamic electronic polarizability function which somewhat deviates from that expected in an ordinary Landau-Fermi liquid. In particular, their polarizability function shows that the quasi-particle weightage at K_F or the discontinuity in quasi-particle occupation number $n(K)$ at K_F vanishes logarithmically. With this polarizability function they can explain the experimentally observed anomalies and unusual features in various properties like electrical resistivity, optical conductivity, NMR relaxation rate etc. in the normal phase, they claim. Their dielectric function shows a regime of dynamic attraction between the carriers extending over an energy scale of $\pm \sim 0.5$ eV, lending support to their earlier conjecture of CT-exciton mediated pairing. The calculated values of T_c and $\frac{2\Delta(0)}{kT_c}$ by Eliashberg scheme from this dielectric function also seem to be quite close to the experimental values.

3. Further theoretical developments

We have taken up several theoretical research projects to analyse various properties of these high T_c oxide systems, both in the undoped as well as in the doped phases. The aspects we are mostly interested in at the moment, are the followings : (i) the mechanism of high-temperature superconductivity within conventional pairing theory (Chaudhury 1990a) and (ii) magnetic property in the insulating undoped phase as well as in the doped phase (Chaudhury 1990b).

3.1. *Mechanism of high-temperature superconductivity within conventional pairing theory :*

We have been involved with the problem of finding out the mechanism which plays the most dominant role in producing the attractive pairing interaction. The magnetic measurements involving D.C. field and neutron scattering experiments seem to show no appreciable spin fluctuations in the doped phase (Tanaka *et al* 1987, Endoh *et al* 1988, Birgeneau *et al* 1988). We, therefore, have been probing the possibility of obtaining a high transition temperature principally from charge fluctuation induced attraction (Chaudhury 1990a).

As a first step, we have investigated the possible role of combined mechanisms of phonon and CT-exciton in producing high T_c in $La_{1.85}Sr_{0.15}CuO_4$, following the suggestion of Rice and Wang (Rice and Wang 1987, Jha 1988).

We use the dielectric function approach to calculate the effective interaction between two carriers, relevant for pairing. As a trial model, our longitudinal dielectric function $\varepsilon(\bar{q}, \omega)$ contains the polarizabilities corresponding to the phonon and CT-exciton modes observed in infra-red (IR) absorption experiment on polycrystalline sample (Rice and Wang 1987, Herr *et al* 1987) as shown in Figure 1 and in addition it also contains the Thomas-Fermi static polarizability appropriate to an electron (hole) gas. In the Random Phase Approximation (RPA) $\varepsilon(\bar{q}, \omega)$ appropriate to our model is given by

$$\varepsilon\left(\vec{q}, \omega\right) = 1 + \frac{q_s^2}{q^2} + 4\pi\chi\,(q \to 0, \omega) \tag{1}$$

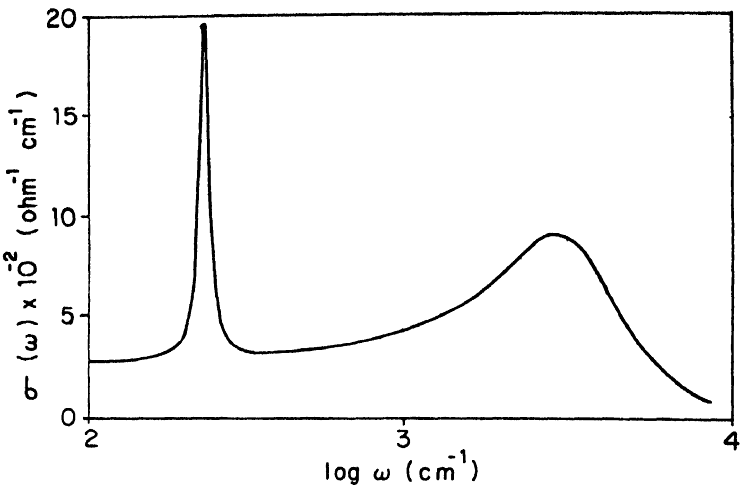

Figure 1. Double-Lorentzian fit to the IR-conductivity date of Herr *et al* (1987)

where $\left(\frac{1}{4\pi}\right)\frac{q_s^2}{q^2}$ is the usual static Thomas-Fermi polarizability for a 3-dimensional (3-D) electron (hole) gas, q_s being the Thomas-Fermi screening wave vector and $\chi\ (q \to 0, \omega)$ or $\chi\,(\omega)$ for simplicity is the polarizability part which contains the observed phonon and CT-exciton modes in IR-absorption. We use a 3-D electron (hole) gas model assuming an effective homogeneous medium for simplicity, as the experimental system relevant to IR-absorption experiment was polycrystalline.

From the IR data (Figure 1) we find that $\chi\,(\omega)$ can be parametrized very well by the two-oscillator model in the usual way,

$$\chi\,(\omega) = \frac{B_1^2}{\omega_{1t}^2 - (\omega + i\Gamma_1)^2} + \frac{B_2^2}{\omega_{2t}^2 - (\omega + i\Gamma_2)^2} \tag{2}$$

where B_l^2 and B_2^2 are the oscillator strengths of the two transverse modes (optical phonon mode and CT-exciton mode) with frequencies ω_{1t} and ω_{2t} respectively and Γ_1 and Γ_2 are the respective damping constants. Then we can approximate the IR absorption spectrum by a Double-Lorentzian form :

$$(\mathrm{Real}\ \sigma\,(\omega)) \simeq \frac{\frac{B_1^2\Gamma_1}{2}}{(\omega_{1t} - \omega)^2 + \Gamma_1^2} + \frac{\frac{B_2^2\Gamma_2}{2}}{(\omega_{2t} - \omega)^2 + \Gamma_2^2} \tag{3}$$

The parameters $B_1, \Gamma_1, \omega_{1t}, B_2, \Gamma_2$ and ω_{2t} can be extracted from the IR data by using the above equation. Now neglecting damping for simplicity the eq. (1) becomes,

$$\varepsilon\left(\bar{q}\,,\,\omega\right) = \varepsilon_C\left(\bar{q}\right) + \frac{4\Pi B_1^2}{\omega_{1t}^2 - \omega^2} + \frac{4\Pi B_2^2}{\omega_{2t}^2 - \omega^2} \tag{4}$$

where $\varepsilon_C\left(\bar{q}\right) = 1 + \dfrac{q_s^2}{q^2}$

Inverting eq. (4) we can also obtain

$$\frac{1}{\varepsilon\left(\bar{q}, \omega\right)} = \frac{1}{\varepsilon_C(q)} + \frac{\dfrac{f_1(q)}{\varepsilon_C(q)}}{\omega^2 - \Omega_1^2(q)} + \frac{\dfrac{f_2(q)}{\varepsilon_C(q)}}{\omega^2 - \Omega_2^2(q)} \tag{5}$$

where $\Omega_1^2(q)$ and $\Omega_2^2(q)$ are the frequencies of the two longitudinal modes and $f_1(q)$ and $f_2(q)$ are the respective oscillator strengths. They are given by the following expressions :

$$\Omega_1^2(q) = \frac{1}{2}\left(\omega_{1t}^2 + \omega_{2t}^2 + \tilde{B}_1^2(q) + \tilde{B}_2^2(q)\right) - \frac{1}{2}\left[\left(\omega_{1t}^2 - \omega_{2t}^2 + \tilde{B}_1^2(q) - \tilde{B}_2^2(q)^2 + 4\tilde{B}_1^2(q)\,\tilde{B}_2^2(q)\right)\right]^{\frac{1}{2}}$$

$$\Omega_1^2(q) = \frac{1}{2}\left(\omega_{1t}^2 + \omega_{2t}^2 + \tilde{B}_1^2(q) + \tilde{B}_2^2(q)\right) + \frac{1}{2}\left[\left(\omega_{1t}^2 - \omega_{2t}^2 + \tilde{B}_1^2(q) + \tilde{B}_2^2(q)^2 + 4\tilde{B}_1^2(q)\,\tilde{B}_2^2(q)\right)\right]^{\frac{1}{2}} \tag{6}$$

and $$f_1(q) = \frac{\left(\Omega_1^2(q) - \omega_{1t}^2\right)\left(\Omega_1^2(q) - \omega_{2}^2\right)}{\left(\Omega_1^2(q) - \Omega_2^2(q)\right)}$$

$$f_2(q) = \frac{\left(\Omega_2^2(q) - \omega_{1t}^2\right)\left(\Omega_2^2(q) - \omega_{2t}^2\right)}{\left(\Omega_2^2(q) - \Omega_1^2(q)\right)} \tag{7}$$

where $\tilde{B}_1^2(q) = \dfrac{B_1^2}{\epsilon_c(q)}$

and $\tilde{B}_1^2(q) = \dfrac{B_2^2}{\epsilon_c(q)}$

The effective interaction between two carriers is given by

$$V_{eff}(q,\omega) = \varepsilon^{-1}(q,\omega)\ V_0(q) \tag{8}$$

where $\quad V_0(q) = \dfrac{4\pi e^2}{q^2}$ is the bare interaction.

The 1st term in the right hand side (rhs) of eq. (5) produces a static screened Coulomb repulsion, the 2nd term produces a screened dynamic attraction for $\omega^2 < \Omega_1^2$ and the 3rd term produces a screened dynamic attraction for $\omega^2 < \Omega_1^2$. Following the usual approach of BCS, we take a 3-square well model (Ginzburg and Kirzhnits 1972, 1982) where the repulsive Coulomb interaction well has range from $-\omega_F$ to $+\omega_F$, the first attractive interaction well has range from $-\omega_1$ to $+\omega_1$ and the second attractive interaction well has range from $-\omega_2$ to $+\omega_2$, where,

$$\omega_1^2 \simeq \Omega_1^2\ (q = 2K_F)$$

and

$$\omega_1^2 \simeq \Omega_2^2\ (q = 2K_F)$$

From eq. (5) we can obtain the expressions for the two attractive coupling constants \B(λ_1 and λ_2), and the repulsive Coulomb coupling constant (μ_C) by calculating the effective average interaction of two conduction electrons close to the Fermi-surface.

They are given by :

$$\left.\begin{aligned} \lambda_i &= \frac{q_s^2}{4K_F^2}\int_0^{2k_F}\frac{dq}{q}\ \frac{1}{\varepsilon_C(q)}\ \frac{f_i(q)}{\Omega_i^2(q)} \\ \mu_C &= \frac{q_s^2}{4K_F^2}\int_0^{2k_F}\frac{dq}{q}\ \frac{1}{\varepsilon_C(q)} \end{aligned}\right\} \tag{9}$$

This gives

$$\mu_C = \frac{q_s^2}{8K_F^2}\ \ln\left(1 + \frac{4K_F^2}{q_s^2}\right) \tag{9a}$$

As an approximation, we write

$$\lambda_i \simeq \left\langle \frac{f_i(q)}{\Omega_i^2(q)} \right\rangle \mu_C \qquad \text{for } i = 1, 2 \tag{10}$$

where $\langle\ \rangle$ denotes the $\vec{q}$-space average.

Again realising that for BCS-like superconductivity the interaction with momentum transfers $\hbar q \simeq \hbar K_F$ contribute most substantially to the attractive coupling constant, we further approximate eq. (10) by

$$\lambda_i = \frac{f_i\left(q \simeq K_F\right)}{\Omega_i^2\left(q \simeq K_F\right)}\,\mu_C \qquad \text{for } i = 1, 2 \tag{10a}$$

We now use 3-well BCS equation for T_c (Ginzburg and Kirzhnits 1972, 1982)

$$T_C = 1.13\theta_1 \exp\left(\frac{-1}{g_{eff}}\right) \tag{11}$$

where $\theta_1 = \dfrac{\hbar\omega_1}{\kappa}$, provided $\omega_1 \le \omega_2 \le \omega_F$

and

$$g_{eff} = \lambda_1 - \frac{\left(\mu_c - \lambda_2^*\right)}{1 + \left(\mu_c - \lambda_2^*\right) Z_C} \tag{11a}$$

with

$$\lambda_2^* = \frac{\lambda_2}{\left[1 - \lambda_2 \ln\left(\dfrac{\omega_2}{\omega_F}\right)\right]}$$

$$Z_C = \ln\left(\frac{\omega_F}{\omega_1}\right) \tag{11b}$$

The numerical values of various parameters occurring in the above equations, obtained mostly from the IR data (Herr *et al* 1987, Rice and Wang 1987) and other measurements (Hundley *et al* 1987) are as follows :

$\omega_{1t} = 240\ \text{cm}^{-1}$; $\omega_{2t} = 3 \times 10^3\ \text{cm}^{-1}$; $n = 6 \times 10^{21}\ \text{cm}^{-3}$; $E_F = 1.2$ eV ;

$q_s^2 = 150 \times 10^{14}\ \text{cm}^{-2}$; $K_f^2 \simeq 31 \times 10^{14}\ \text{cm}^{-2}$; $B_1 \simeq 400\ \text{cm}^{-1}$; $B_2 \simeq 4000\ \text{cm}^{-1}$.

These give, $\varepsilon_{\text{undoped}}(0, 0) = 1 + 4\pi\chi(0) = 43$

which very well agrees with the value obtained experimentally for La_2CuO_4 (Ganguly 1988) from other experiments.

We also get, $\theta_1 \simeq 360$K ; $\omega_2 \simeq \omega_F$ and $\omega_1 < \omega_2 \simeq \omega_F$.

Thus $\lambda_2^* \simeq \lambda_2$ from eq. (11b).

Also combining the eqs. (9), (7) and (6), $\lambda_2 < \mu_C$.

However, to be most optimistic, we assume,

$$\lambda_2 \simeq \mu_C$$

Then from the eqs. (11) and (11a), we get an effective L-well BCS equation for T_c, viz.

$$T_C \simeq 1.13\theta_1 \exp\left(\frac{-1}{\lambda_1}\right) \tag{12}$$

Making use of the eqs. (10a), (7) and (6) we calculate λ_1 and find

$$\lambda_1 \simeq 0.05\,\mu_C$$

Again for the jellium model in consideration,

$\mu_C \simeq 0.5$, from the eq. (9a).

This gives $\lambda_1 \simeq 0.025$.

Then from eq. (12) we find T_C to be vanishingly small.

Thus we find that only the combined mechanism of phonon and CT-exciton within the usual BCS pairing scheme and BCS approximation cannot account for the observed T_C (~ 36K) of polycrystalline $La_{1.85}Sr_{0.15}CuO_4$ (Bednorz and Müller 1986).

We later tried to incorporate some of the effects of very short coherence length (~ 10-12 Å) claimed to have been observed in this system (Cava *et al* 1987) by considering the case where the whole Fermi sphere takes part in pairing. We go beyond the usual BCS approximation of replacing $N(\xi')$ in the integrand by $N(0)$, i.e. the carrier density of states at the Fermi level, in the linearised BCS gap equation. We, however, find that the correction to the usual BCS equation for T_c is of the order of 10^{-5}. Thus our earlier conclusion reached regarding the smallness of magnitude of T_c remains unchanged.

We later tried to incorporate the effect of exchange-correlation enhanced spin fluctuations in our model, following our earlier work (Chaudhury and Jha 1984). We find that the enhancement in electronic spin susceptibility χ_m in the normal phase, if at all it is appreciable in this system, does not change the overall conclusion reached regarding the magnitude of T_c ; although it may change the signs of the individual coupling constants produced by different excitations. We must admit, however, that the calculation of coupling constants has been done somewhat crudely ; the $\vec{q}$-space averaging should probably be done more accurately by performing the integrations numerically.

It may be mentionworthy that the data for $\sigma(\omega)$ from the experiments of Herr *et al* for polycrystalline sample has no appreciable weightage for Drude-like features, whereas most of the experiments performed later on single crystals exhibit a very strong Drude-like feature (Thomas *et al* 1988, Batlogg 1990) besides a hump at finite ω. The electronic plasmons, thus missed in our trial model may play an important role in producing sufficiently strong attractive interaction. This investigation is currently in progress. In any case, the interesting suggestion of Rice and Wang involving only the combined mechanism of phonon and CT-exciton seems to be inadequate for yielding the high value of T_c ; however it does give superconductivity.

In this work we have treated the system as a 3-dimensional homogeneous system.

The possible complications arising from the presence of detailed layered-structure (Jha 1987) relevant particularly for single crystals, will be probed in a future work.

Our recent calculation using the single crystal data for $\sigma(\omega)$ containing the prominent Drude-peak along with a finite-ω hump shows the possibility of obtaining a very high value of T_c from the exchange of charge fluctuations (Chaudhury 1991). In this work we have also incorporated quasi-two dimensional nature of the electronic systems seen in some of the oxides.

3.2 *Magnetic property in the insulating and doped phases :*

We had been originally motivated to study the magnetic correlations in these Copper-oxide systems (Chaudhury 1990b) primarily because of various conjectures (Anderson 1987a, 1987b) regarding the importance of quantum fluctuations in 2-D Heisenberg antiferromagnet with $S = \frac{1}{2}$.

Various numerical calculations (Horsch 1988) performed recently have suggested the ground state of 2D spin $\frac{1}{2}$ Heisenberg antiferromagnet (HAF) to be predominantly Neel-like with some very small contribution from states containing a few domain-wall like defects, in contrary to the RVB configuration conjectured by Anderson (Anderson 1987a, 1987b). There have also been analytic and semi-analytic-semi-numerical calculations (Auerbach and Arovas 1988, Chakravarty *et al* 1988, Tyc *et al* 1989) which show that the spin dynamics of 2D $S = \frac{1}{2}$ QHAF is essentially spinwave like. This is in contradiction with the results of inelastic neutron scattering experiments on single crystals of La_2CuO_4, (Endoh *et al* 1988, Yamada *et al* 1989) which show a pronounced central peak in dynamic structure factor $S(\vec{q}, \omega)$ even for $\left|\vec{q}_{2D} - \vec{q}_{2B}\right| \simeq \kappa$, κ being the inverse correlation length, in a temperature regime (above the Neel temperature T_N of the 3-D system) (see Figure 2) where the magnetic correlations are seen to be entirely two-dimensional with $\xi \sim 200$ Å at T = 300K. This discrepancy between the theoretical results and experimental observation motivated us further to have a closer look at the experimental system La_2CuO_4 (Chaudhury 1990b).

We notice the fact that the intra-layer antiferromagnetic coupling is not purely isotropic Heisenberg-like but rather has a small XY-anisotropy. The actual spin Hamiltonian relevant to La_2CuO_4 is (Endoh *et al* 1988).

$$H = -J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j + J_A \sum_{<i,j>} S_i^Z \cdot S_j^Z - J' \sum_{<i,k>} \vec{S}_i \cdot \vec{S}_k \tag{13}$$

where

J = isotropic part of the intra-layer coupling

J_A = anisotropic part of the intra-layer coupling

J' = inter layer coupling

This kind of Hamiltonian has been used earlier to describe other layered systems (Hirakawa *et al* 1982, 1983) such as K_2CuF_4 (a ferromagnet with $S = \frac{1}{2}$ which also has a small XY — anisotropy) and K_2NiF_4 $\left(S = \frac{1}{2}\right.$ ferromagnet which has an Ising-anisotropy$\left.\right)$. It was found by Hirakawa and coworkers (Hirakawa *et al* 1982, 1983) from neutron scattering data for K_2CuF_4 that in a temperature regime $T_1 \le T \le T_2$,

where

$$\left.\begin{array}{l} \xi(T_1) \sim \left(\dfrac{|J|}{|J'|}\right)^{\frac{1}{2}} \\ \xi(T_2) \sim \left(\dfrac{|J|}{|J_A|}\right)^{\frac{1}{2}} \end{array}\right\} \quad \text{in unit of } a \tag{14}$$

the system behaves as a 2-D XY-like ferromagnet, as expected from the considerations of energetics of spins confined within a correlated region of area ξ^2. Further, it was found that

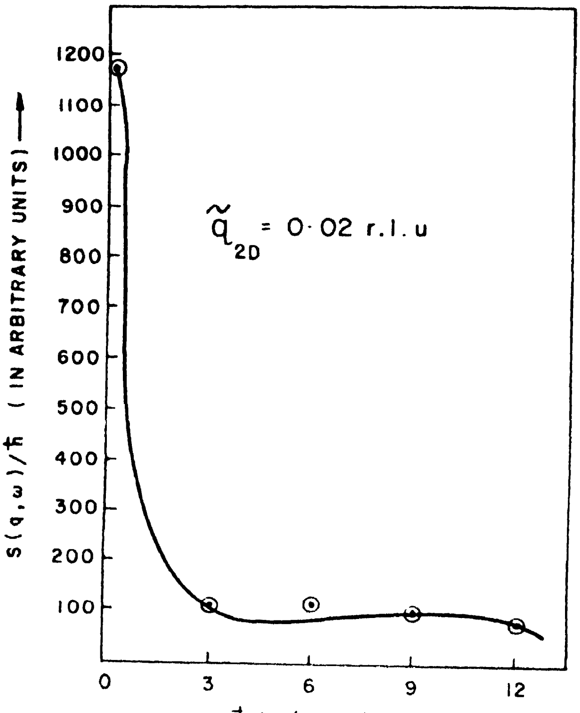

Figure 2. Convoluted Dynamical Structure Function S $(\tilde{q}, \omega)$ VS ω from Neutron Scattering Experiment with scans across the 2D rod

the extracted static correlation function in this temperature regime obeys a Kosterlitz-Thouless (KT) predicted form (Kosterlitz and Thouless 1973) with a value of T_{KT}

(Kosterlitz-Thouless transition temperature) situated below this regime,

$$\kappa^{-1}(T) \equiv \xi(T) = \xi_0 \exp\left(-bt^{-\frac{1}{2}}\right) \tag{15a}$$

where, $\xi_0 = 0\,(a)$, a being the lattice spacing

$b \simeq 1.5 - 1.7$

$$t \equiv \frac{(T - T_{KT})}{T_{KT}}$$

with
$$S_{q_{2D}} \propto \frac{1}{\left(\kappa^2 + q_{2D}^2\right)^{1-\frac{\eta}{2}}} \; : \; \eta \simeq 0.25 \tag{15b}$$

For K_2CuF_4 it turned out that $T_1 = 6.6K$, $T_2 = 7.3K$; $T_{KT} = 5.5K$. Moreover we found that this fitted value of T_{KT} for the normalized Z component coupling $\lambda_Z = 0.99$ obeys the Monte Carlo-Molecular Dynamics (MCMD) result (Kawabata and Bishop 1982, 1986) connecting T_{KT} with a finite λ_Z and T_{KT} with $\lambda_Z = 0$ corresponding to the purely XY case, viz.,

$$T_{KT}(\lambda_Z = 0.99) = 0.67\, T_{KT}(\lambda_Z = 0) \tag{16a}$$

where
$$T_{KT}(\lambda_Z = 0) \simeq \frac{\pi}{2} \left|J\right| S^2 \tag{16b}$$

is the classical result of Kosterlitz and Thouless (Kosterlitz and Thouless 1973). Besides, the study of spin dynamics by Hirakawa and coworkers (Hirakawa *et al* 1983) has shown a very prominent central peak for $S\left(\vec{q}, \omega\right)$ in this regime $T_1 \le T \le T_2$ for $\left|\vec{q}_{2D} - \vec{q}_{ZB}\right| \gg \kappa$ but of course with some other restrictions on $\left|\vec{q}\right|$ (Hirakawa *et al* 1983). This observation further supported the idea of the presence of topological excitation viz. mobile free vortices and anti-vortices for $T > T_{KT}$ which are known to lead to a pronounced central peak (Mertens *et al* 1987, Huber 1982) in $S\left(\vec{q}, \omega\right)$. Thus in this temperature regime K_2CuF_4 seems to behave quite well as an ideal KT system.

We tried to apply the same phenomenological approach to the case of La_2CuO_4, spin $\frac{1}{2}$ antiferromagnet with $\lambda_Z \simeq 0.9999$. We indeed find a regime $T_1 \le T \le T_2$ where experimentally obtained $\xi(T)$ obeys the KT predicted form very well with

$$S_{q_{2D}} \propto \frac{1}{\left\{\kappa^2 + \left(q_{2D} - q_{ZB}\right)^2\right\}^{1-\frac{\eta}{2}}}$$

and $T_1 = 350K$; $T_2 = 400K$: $T_{KT} = 275K$ (see Figure 3). We would like to point out that η may be very small here.

Thus again we can very easily explain the central peak in $S(q, \omega)$ observed by Shirane and coworkers at $T = 300$K as a manifestation of the spin dynamics induced by the mobility of unbound vortices and antivortices. It may be worthwhile to mention that in the case of La_2CuO_4, the vortices and antivortices are to be constructed from staggered spin vectors because of the presence of strong antiferromagnetic correlations. We, however, find one anomaly in the case of La_2CuO_4. If we use the results from simulation and renormalization group calculations (Kawabata and Bishop 1982, 1986, Khokhlachev 1976) for the asymptotic case viz.,

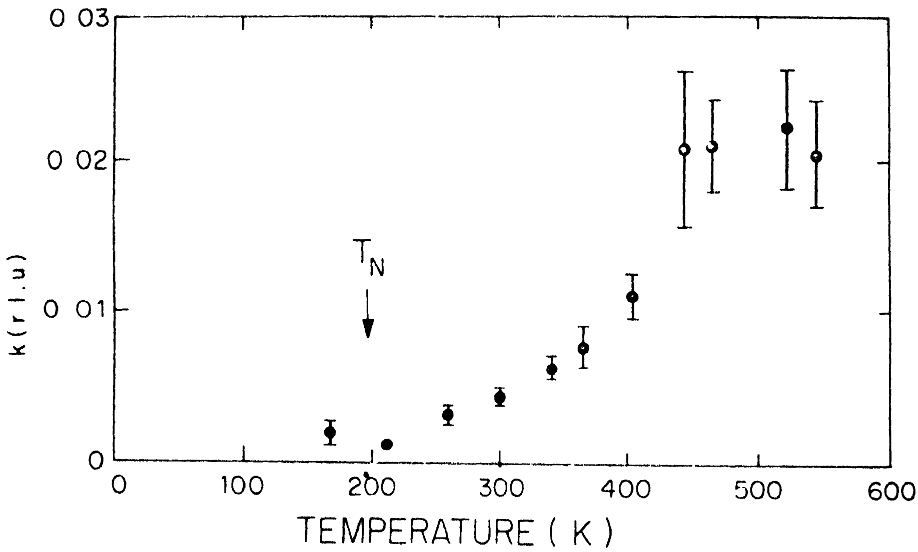

Figure 3. Inverse correlation Length vs Temperature for La_2CuO_4

$$T_{KT}(\lambda_z \to 1) \simeq \frac{T_{KT}(\lambda_z = 0)}{\ln(1-\lambda_z)^{\frac{1}{2}}} \tag{16c}$$

then we get

$$T_{KT}(\lambda_z = 0) = 1266\text{K}.$$

If we use the classical result of KT, then,

$$T_{KT}(\lambda_z = 0) \sim 319\text{K}$$

Thus the two results differ by a factor of 4 ! This seems to show that La_2CuO_4 in the above temperature regime displays an anomalous KT behaviour. This may imply that for La_2CuO_4 the non-ideal behaviour of the KT plasma viz. vortex-vortex (antivortex-antivortex) interaction and spinwave-vortex (antivortex) coupling are quite important.

Detailed phenomenological comparison of $S\left(q, \omega\right)$ for both La_2CuO_4 and K_2CuF_4 with the results of Bishop and coworkers (Mertens *et al* 1987) for an ideal KT plasma is in progress. This will help in understanding the origin of the anomaly in the case of La_2CuO_4. The questions whether quantum fluctuation can enhance even the free vortex density (Betts *et al* 1981) besides increasing the bound pair density and how important the quantum fluctuation is in the case of La_2CuO_4, are also expected to be resolved from this study of $S\left(q, \omega\right)$. We also plan to carry out a full MCMD simulation for the model appropriate to La_2CuO_4, to understand all these aspects more clearly.

It may be mentioned that these vortices look very similar to the topological solitons called 'merons' proposed later in RVB theory (Anderson *et al* 1988) and may form a charged 'anyon' gas in the doped phase of the system, which can show pairing instability leading to an "exotic kind of superconductivity", as conjectured by Witten and coworkers (Chen *et al* 1989). Search for a topological term in the effective action corresponding to our model is also in progress (Paul and Chaudhury 1991).

From neutron scattering data one sees that in the doped phase viz. $La_{2-x}Sr_xCuO_4$ (Birgeneau *et al* 1988, Thurston *et al* 1989) the static spin-spin correlation function shows a strong q_Z dependence above T_N and ξ_{2D} shows a split peak structure with a dip at q_{ZB} in contrast to the peak at q_{ZB} seen in the undoped phase. This probably implies that vortices even if they survive in the doped phase, have to get highly transformed. T_N of the 3D system drops very rapidly with doping and vanishes at a doping concentration of about 2.5% (Müller 1988) ; ξ_{2D} falls very rapidly with increase in doping concentration in the 'spin glass-like' phase, appearing after the destruction of long range order and reaches a low value of ~ 10 Å for about 7% doping level. This shows the high degree of frustration the spin suffer in the doped phase. $S\left(q, \omega\right)$ in the doped phase shows the spin fluctuations to be predominantly dynamic at high temperature and quasi-elastic at low temperature, implying a some sort of freezing of the spins. The integrated intensity $\int S\left(q, \omega\right) dq\, d\omega$ is however, the same in the doped and undoped phases and gives the local moment. All these properties remain intact in the superconducting phase, as well.

These phenomenological aspects may bring out the limitations of simple *t-J* model (Anderson 1987a, 1987b, Baskaran 1988) to describe the doped phase and imply the importance of the *t-J-J* type of models (Baskaran 1988) which include the additional 3-site terms corresponding to pair-hopping processes. These terms may also be useful for characterising the 'spin glass phase' properly in terms of new order parameter similar to "chirality" (Baskaran 1989a, 1989b). Besides, any serious calculation of superconducting and magnetic order parameters should satisfy the phenomenological constraints mentioned above. The investigation with this *t-J-J* model is currently in progress. Our recent work on the'coherent bond states' in the general context of the Hubbard model also throws some light on these issues (Chakrabarti and Chaudhury 1991 , Chaudhury 1990c).

4. Some Experimental Studies

Many of the known high temperature superconductors have been prepared and shown to be

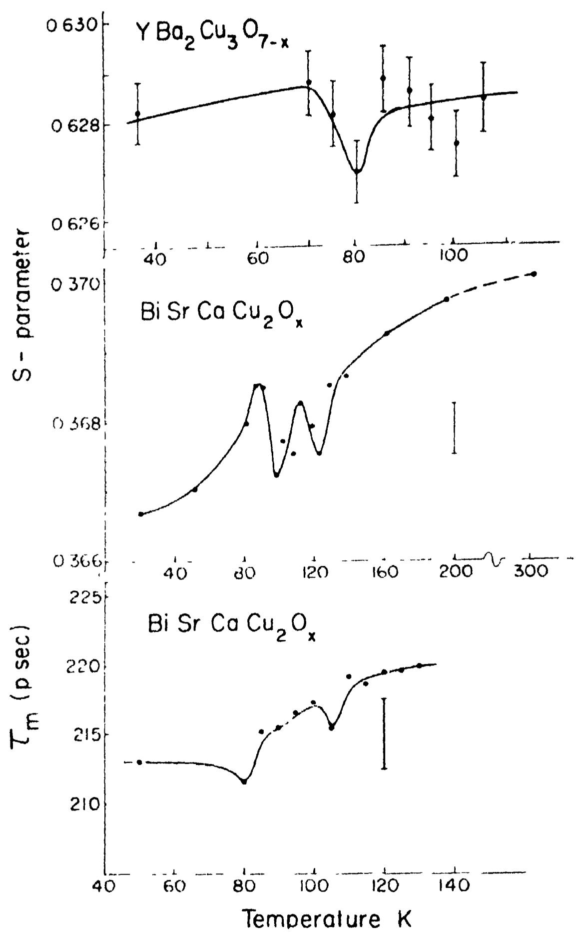

Figure 4. Positron annihilation data showing anomalies near the transition temperature. The S-parameter in the Doppler broadening of the annihilation shows one anomaly in 1 2 3 compound and two anomalies in Bi-Sr-Ca-Cu-O The mean life time of the positron in the later also show two anomalies correlated with the transition temperatures

superconducting by the collaborating experimentalists at the Saha Institute of Nuclear Physics (SINP). $La_{1.8}Sr_{0.2}CuO_4$ samples were prepared from mixing stoichiometric amount of La_2O_3, CuO and SrO and sintering and grinding. Appropriate amounts of Y_2O_3, $BaCO_3$ and CuO were mixed and heated suitably, and pressed into pellets and sintered. Bi_1 Sr_1 Ca_1

Cu_2O_x were prepared by mixing appropriate amounts of bismuth oxide, copper oxide, strontium carbonate and calcium carbonate and heating in oxygen, grinding and annealing and cooling in oxygen. Also Tl-Ca-Ba-Cu-O samples were prepared with necessary precaution for poisonous Tl. X-ray diffraction was used to confirm the correct structure.

DC resistance measurement was done using the conventional four probe technique to find the temperature of zero resistance. The dc magnetic measurements have been done with a vibrating sample magnetometer to detect the Meissner signal. Other physical measurements like Hall effect or thermopower have been done to measure carrier concentration and the sign of charge of the current carriers.

One technique which might have a bearing on the mechanism of transition is the positron annihilation technique. A positron in a solid at liquid nitrogen temperature or above interacts with an electron by the Coulomb force and annihilates with it. Because of its positive charge, it keeps away from ion cores and its interaction with lattice vibrations is negligible. In conventional BCS superconductors the ratio $\frac{\Delta}{E_F}$, the energy gap by the Fermi energy, is a measure of the change brought about by the phonons and is essentially not seen by positrons. If, however, the mechanism of superconductivity in the ceramic superconductors is purely electronic, there is a chance that positron annihilation will show some signature.

Position lifetime and Doppler-broadened positron lineshape studies have been done on the high temperature superconductors. The experimental apparatus and its calibration are described in Mandal *et al* (1988). Several lifetimes are shown, and the data are complex. The line-shape is described by the shape parameter S, which is the ratio of the area under the central channels around the peak to the total area under the annihilation line. Because of the thermal expansion of the solid, the S parameter should show a steady rise with temperature.

The line shape studies were carried out on $YBa_2Cu_3O_{7-x}$ and $HoBa_2Cu_3O_{7-x}$ between 36 and 105 K. Similarly the $BiSrCaCu_2O_x$ was studied between 23 and 300 K. However, it shows a dip in $YBa_2Cu_3O_{7-x}$ around 84 K, the transition temperature determined by resistivity measurements ; similarly there is a dip at 91 K, the transition temperature, in Ho $Ba_2Cu_3C_{7-x}$. The S parameter in $BiSrCaCu_2O_x$ shows two anomalies at the two transition temperatures 105 and 84 K, as observed in resistance measurements. Because of the large error bars, one may doubt if there are two dips or just one dip around 105 to 80 K. Detailed analysis of the lifetimes has, however, confirmed that the mean lifetime τ_m does have two anomalies in agreement with those apparent in the S parameters (because of several lifetimes and possible presence of defects, the mean-lifetime τ_m is perhaps the only significant quantity).

Quantitatively, the anomalies in the S parameters are small-fraction of a percent. On approaching the transition from the high temperature side S parameter decreases and the line broadens ; below the transition the S parameter goes up, showing that the low momentum components in the central channels gather more than usual intensity. They are indicative of an electronic mechanism but do not prove that. One can definitely conclude that the mechanism is the same in Y-Ba-Cu-O and Bi-Sr-Ca-Cu-O compounds.

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References

Aharony A, Birgeneau R J and Kastner M A 1988 *Int. J Mod Phys* **B1** 649
Alascio B, Gagliano E, Avignon M and Balseiro C A 1988 *Int J Mod Phys* **B1** 931
Anderson P W 1987a *Science* **235** 1196
—— 1987b *Phys Rev Lett* **59** 2407
—— 1973 *Mat Res Bul* **8** 153
Anderson P W, Baskaran G, Zhou Z and Hsu T 1987 *Phys Rev Lett* **58** 2790
Auerbach A and Arovas D P 1988 *Phys Rev Lett* **61** 617
Anderson P W, John S, Baskaran G, Doucot B and Liang S D 1988 *Princeton Preprint*
Bednorz J G and Muller K A 1986 *Z Phys* **B64** 189
Bulaevskii L N, Dolgov O V and Ptitsyn M O 1988a *Phys Rev* **B38** 11290
Bulaevskii L N *et al* 1988b *Science and Technology Preprint*
Birgeneau R J *et al* 1988 *Preprint*
Baskaran G 1990 Talk delivered at the 'International Conference on superconductivity» held at Bangalore.
Baskaran G 1989a *NSF/ITP Preprint*
—— 1989b *Talk delivered at the Miniworkshop on Strongly-correlated Electron Systems* (ICTP, Trieste)
Bianconi A *et al* 1988 *Int. J. Mod Phys* **B1** 1151
Batlogg B 1990 *Private Communications*
Baskaran G 1988 *Int. J Mod Phys* **B1** 539
Betts D D, Salevsky F C and Rogiers J 1981 *J Phys* **A14** 531
Capponi J J *et al* 1987 *Europhys Lett* **3** 1301
Carbotte J P 1990 *Rev Mod Phys* **62** 4 1027
Chu P *et al* 1987 *Phys Rev Lett* **58** 405
Chakraverty B K, Feinberg D, Hang Z and Avignon M 1987 *Solid State Commun* **64** 1147
Chen Y H, Wilczek F, Witten E and Halperin B I 1989 *IAS Preprint*
Cava R J *et al* 1987 *Phys Rev Lett* **58** 1676
Chaudhury R 1990a *SNBNCBS Preprint*
—— 1990b *Talk delivered at the Workshop on Physics of Ceramic Superconducting Materials* (SINP, Calcutta)
Chaudhury R 1990c SNBNCBS Preprint
—— 1991 In Preparation.
Chaudhury P and Jha S S 1984 *Pramana J Phys* **22** 431
Chakrabarti J and Chaudhury R 1991 *Mod Phys Lett.* **B5** 1525
Chakravarty S, Halperin B I and Nelson D R 1988 *Phys. Rev Lett* **60** 1057
Doniach S 1988 *Lecture delivered at the Miniworkshop on Mechanisms for High Temperature Superconductivity* (ICTP, Trieste)
Endoh Y *et al* 1988 *Phys Rev* **B37** 7443
Emery V J 1987 *Phys Rev Lett* **58** 2794

Fazekas P and Anderson P W 1974 *Phil Mag.* **30** 423

Gros C, Joynt R and Rice T M 1987 *Z Physik* **B68** 425

Ginzburg V L and Kirzhnits D A 1972 *Phys Rep* **C4** 344

—— 1982 *High Temperature Superconductivity* (New York · Consultants Bureau)

Ganguly P 1988 *Private Communications*

Herr S L *et al* 1987 *Phys Rev* **B36** 733

Hundley M F, Zettl A, Stacy A and Cohen M L 1987 *Phys Rev* **B35** 8800

Horsch P 1988 *Lecture delivered at the miniworkshop on 'Mechanisms For High Temperature Superconductivity'* (ICTP, Trieste)

Hirakawa K, Yoshizawa H and Ubukoshi K 1982 *J Phys Soc Jpn* **51** 2151

Hirakawa K, Yoshizawa H, Axe J D and Shirane G 1988 *J Phys Soc Jpn* **52** 4220

Huber D L 1982 *Phys Rev* **B26** 3758

Hirsch J E, Tang S, Loh E (Jr.) and Scalapino D J 1988 *Phys Rev. Lett* **60** 1668

Jha S S 1988 *Private Communications*

—— 1987 *Pramāna J Phys* **29** L615

Kulik I O 1988 *International Journal of Modern Physics* **B1** 851

Kamimura H 1987 *Jpn. J Appl Phys 26 Suppl* **26-3** 1092

Kresin Z 1987 *Phys Rev* **B35** 8716

Katayama-Yoshida H *et al* 1988 *Int Mod Phys* **B1** 1273

Kosterlitz J M and Thouless D J 1973 *J Phys C Solid State Phys* **6** 1181

Kawabata C and Bishop A R 1982 *Solid State Commun* **42** 595

—— 1986 *Solid State Commun* **60** 169

Khokhlachev S B 1976 *JETP* **43** 137

Lyons K B, Fleury P A, Schneemeyer L F and Waszczak J V 1988 *Phys Rev Lett* **60** 732

Laughlin R B 1988 *Science* **242** 525

List R S *et al* 1988 *Los Alamos Preprint*

Mattheiss L F 1987 *Phys Rev Lett* **58** 1028

Mahanty J and Das M P 1988 *Preprint*

Majumdar C K and Ghosh D K 1969 *J Math Phys* **10** 1388

Mertens F G, Bishop A R, Wysin G M and Kawabata C 1987 *Phys Rev Lett* **59** 117

Muller K A 1988 *Invited Lecture delivered at the Conference on Towards the Theoretical Understanding of High T_c Superconductors* (ICTP, Trieste)

Mandal P *et al* 1988 *J Phys* **C21** 3151

Paul S and Chaudhury R 1991 In Preparation

Ruvalds J 1987 *Phys Rev* **B35** 8868

Rice M J and Wang Y R 1987 *Phys Rev* **B36** 8794

Schrieffer J R, Wen S G and Zhang S C 1988a *Preprint*

—— 1988b *Phys Rev Lett* **60** 944

Shirane G *et al* 1987 *Phys Rev Lett* **59** 1613

Su Z B, Yu L, Dong J H and Tosatti B 1988 *Z Phys* **B70** 131

Stephan W H, Linden W V D and Horsch P 1988 *Int J Mod Phys* **B1** 1005

Toyazawa Y, Inoue M, Inui T, Okazaki M and Hanamura E 1966a *J Phys Soc Jpn* **21** 208

—— 1966b *J Phys Soc Jpn* **21** 209

Tesanovic Z, Bishop A R, Martin R L and Marris C 1988 *Int J Mod Phys* **B1** 907

Tranquada J M *et al* 1988 *Phys Rev Lett* **60** 156

Thurston T R *et al* 1989 *Phys Rev* **B40** 4585

Tanaka H *et al* 1987 *Talk delivered at the Conference on High Temperature Superconductivity* (ICTP, Trieste)

Thomas G A *et al* 1988 *Phys Rev. Lett.* **61** 1313

Tyč S, Halperin B I and Chakravarty S 1989 *Phys. Rev Lett.* **62** 835

Varma C M, Littlewood P B, Schmitt-Rink S, Abrahams E and Ruckenstein A E 1989 *Phys. Rev. Lett.* **63** 1996

Varma C M and Schmitt-Rink S and Abrahams E 1987 *Solid State Commun.* **62** 681

Weber W 1987 *Phys Rev Lett* **58** 1371

Yamada K *et al* 1989 *Phys Rev* **B40** 4557

Zou Z and Anderson P W 1988 *Phys Rev* **B37** 627